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Geodesic visualization for the Frutos metric

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Overview

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3. Program
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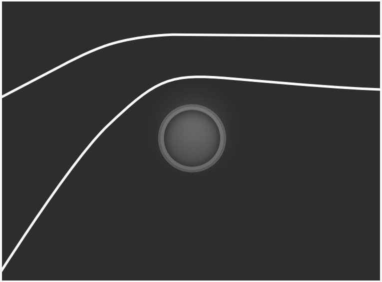
Introduction

Outline of the problem



- Compact/massive objects deform spacetime
- Light is affected; we are interested on the trajectories of photons
- Compare effects for different metrics, specially the Frutos metric

Theoretical framework

General relativity and geodesics

- Metrics describe the curvature of spacetime
- Geodesic equations: set of second order differential equations that contain the derivatives of the metric
- Null geodesics: trajectories of light
- Null geodesics can be used to study:
  - The properties of spacetime described by the metric[1]
  - High resolution gravitational lenses[2]
  - Radiation emitted near a compact object [3]

[1]Hagihara (1930)
[2]Vincent et al. (2011)
[3]Yang et al. (2014), Cunningham (1975)

Astrophysical objects

Objects capable of producing such deformations to spacetime

- Black holes
- Neutron stars
- Galaxies
- Dark matter

Geodesic equations

$$\frac{\mathrm{d}^2 x^\kappa}{\mathrm{d}\lambda^2} = -\Gamma^\kappa_{\mu\nu} \frac{\mathrm{d}x^\mu}{\mathrm{d}\lambda} \frac{\mathrm{d}x^\nu}{\mathrm{d}\lambda}$$

- $\Gamma^\kappa_{\mu\nu}$ are the Christoffel symbols
- λ is an affine parameter
- For null geodesics (light), $ds^2 = 0$

Frutos metric

- Approximate metric with mass, small quadrupole moment (deformation) and rotation
- Kerr has problems when trying to match to an interior solution (e.g. Frutos 2015b)

General structure of the metric

$$ds^2 = g_{tt}dt^2 + 2g_{t\phi}dtd\phi + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{\phi\phi}d\phi^2$$

$$g_{tt} = \frac{e^{-2\psi}}{\rho^2}[a^2 \sin^2\theta - \Delta]$$

$$g_{t\phi} = -\frac{2Jr}{\rho^2}\sin^2\theta$$

$$g_{rr} = \rho^2 \frac{e^{2\chi}}{\Delta}$$

$$g_{\theta\theta} = \rho^2 e^{2\chi}$$

$$g_{\phi\phi} = \frac{e^{2\psi}}{\rho^2}[(r^2 + a^2)^2 - a^2\Delta \sin^2\theta]\sin^2\theta$$

where $\Delta = r^2 - 2Mr + a^2$; $\quad \rho^2 = r^2 + a^2\cos^2\theta$ and if $\psi = \chi = 0$ we get the Kerr metric.

Second order expansions for ψ and χ [4]

$$\psi = \frac{q}{r^3} P_2 + 3\frac{Mq}{r^4} P_2$$

$$\chi = \frac{qP_2}{r^3} + \frac{Mq}{r^4}\left(-\frac{1}{3} + \frac{5}{3}P_2 + \frac{5}{3}P_2^2\right) + \frac{q^2}{r^6}\left(\frac{2}{9} - \frac{2}{3}P_2 - \frac{7}{3}P_2^2 + \frac{25}{9}P_2^3\right)$$

where $P_2 = (3\cos^2\theta - 1)/2$ (Legendre polynomial of second order).

[4] Frutos (2015a)

First order expansions for ψ and χ

If

$$\psi = \chi = \frac{2qM^3}{15}\frac{P_2}{r^3}$$

we obtain the metric described in Frutos et al. (2015b), already tested with our program

Parameters

The parameters of these metrics are

- M, mass
- a, angular momentum parameter
- q, quadrupole moment parameter (small)
- Geometrical units ($G = c = 1$)
- They can be matched to an interior solution[5]

[5] Frutos (2015a)

Previous works

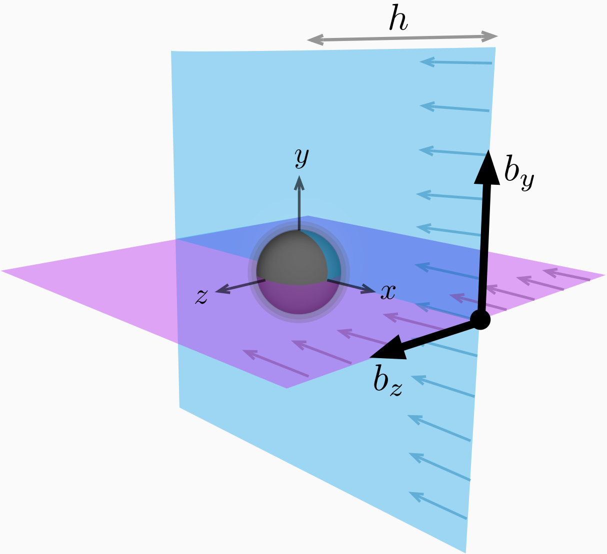
- Analytical study of geodesics; properties of spacetime: Hagihara (1930)
- First computational works: Cunningham (1975), Luminet (1979)
- Semi-analytical integration: Dexter & Agol (2010), Yang (2014)
- Arbitrary numerical metric (3+1 formalism): Vincent et al. (2011)
- HPC / GPU for particular metrics: Müller (2011)
- Use of geodesics for lens reconstruction: Tessore et al. (2015)

Program

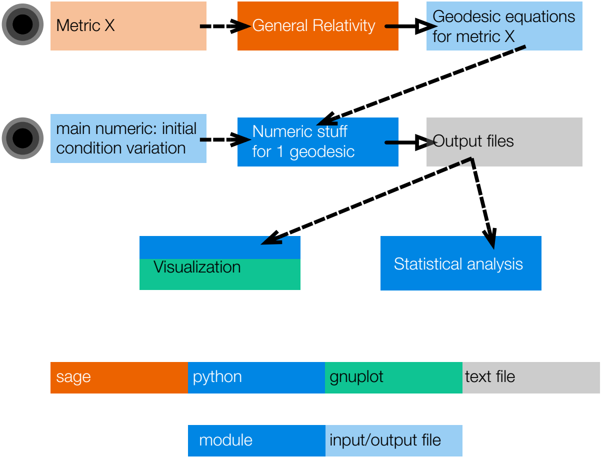
Program

- Takes an analytical metric and computes and solves the geodesic equations
- It *should* work for *any* metric
- Python, Sage, Gnuplot
- So far applied to
    - Schwarzschild
    - Bonnor
    - Kerr
    - Frutos

Initial conditions

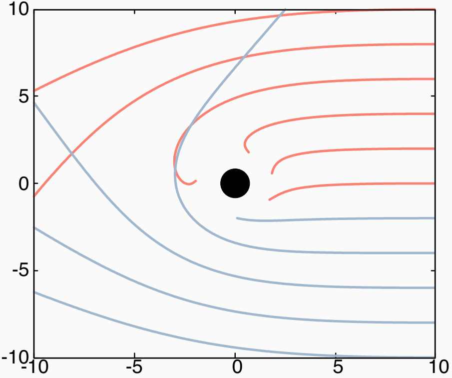


General scheme

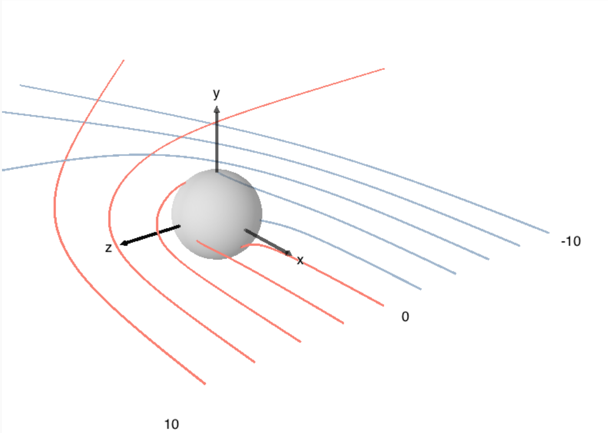


Partial results

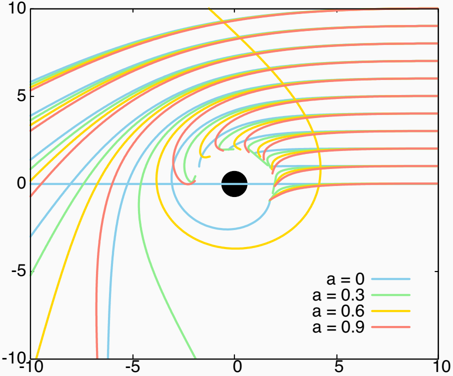
Frutos second order: xy-plane



Frutos second order: xz-plane

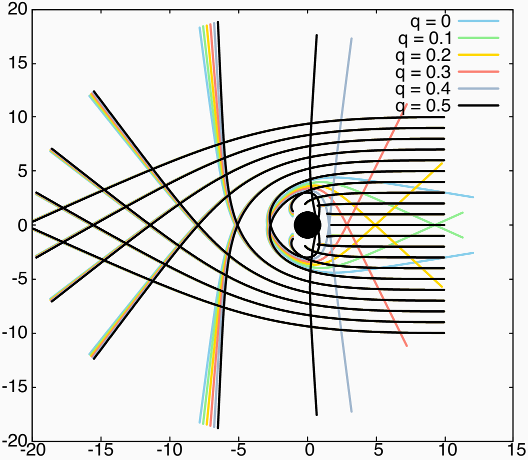


Frutos second order: effect of the angular momentum



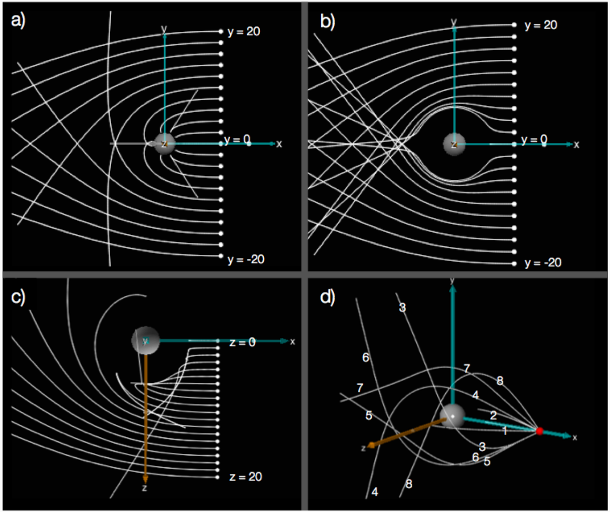
xy plane, $M = 1$, $q = 0.1$

Frutos second order: effect of the quadrupole moment



xz plane, $M = 1$, $a = 0.9$

Bonnor



Future work

Future work

- Extend to massive particles
- Use geodesics to study emission near a compact object for different metrics (output an observable)
- Make comparisons of different metrics; effects of terms
- Writing testing modules, open-source and publish the code in a repository

Main references I

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