Laplace transform

→ `laplace(f(t),t,s);`

(%o2) laplace(f(t), t, s)

1 Linearity

→ `laplace(f(t)+a·g(t),t,s);`

(%o3) a laplace(g(t), t, s) + laplace(f(t), t, s)

2 Exponential attenuation

→ `assume(a>0);`

(%o4) [a > 0]

→ `laplace(exp(−a·t)·f(t),t,s);`

(%o5) laplace(f(t), t, s + a)

3 Scaling

→ `ratsimp(laplace((a·t + a^2·t^2),t,s));`

(%o8) $\frac{a\, s + 2\, a^2}{s^3}$

→ `ratsimp(1/a·laplace((t+t^2),t,s/a));`

(%o9) $\frac{a\, s + 2\, a^2}{s^3}$

4 Derivatives

→ `diff(f(t),t);`

(%o10) $\frac{d}{d\, t} f(t)$

→ `laplace(diff(f(t),t),t,s);`

(%o11) s laplace(f(t), t, s) − f(0)

→ `laplace(diff(f(t),t,2),t,s);`

(%o12) $-\left(\left.\frac{d}{d\, t} f(t)\right|_{t=0}\right) + s^2 \text{ laplace}(f(t), t, s) - f(0)\, s$

→ `integrate(exp(−s·t)·diff(f(t),t),t,0,inf);`

(%o13) $\int \%e^{-(s\, t)} \left(\frac{d}{d\, t} f(t)\right) d\, t$

→ `load("bypart.mac");`

(%o14) /opt/homebrew/Cellar/maxima/5.47.0_19/share/maxima/5.47.0/share/integration/bypart.mac

→ `expr1: byparts(diff(f(t),t)·exp(−s·t),t, exp(−s·t), diff(f(t),t));`

expr1 $s \int \%e^{-(s\, t)} f(t)\, d\, t + \%e^{-(s\, t)} f(t)$

→ `subst(inf,t,part(expr1,2)) − subst(0,t,part(expr1,2)) ;`

(%o16) $f(\infty)\, \%e^{-(\infty\, s)} - f(0)$

we assume that the first term goes to zero.

5 Integrals

→ `g(t):= a·t^2 − t;`

(%o17) $g(t) := a\, t^2 - t$

→ `ratsimp(laplace(integrate(g(t),t),t,s));`

(%o18) $-\left(\frac{s-2\,a}{s^4}\right)$

→ ratsimp(laplace(g(t),t,s)/s);

(%o19) $-\left(\frac{s-2\,a}{s^4}\right)$

6 Partial fractions for the inverse Laplace transform

→ originalexpr: s/(s^2 – k^2);

originalexpr $\frac{s}{s^2-k^2}$

→ expr2: partfrac(s/(s^2 – k^2),s);

expr2 $\frac{1}{2\,(s+k)}+\frac{1}{2\,(s-k)}$

Inverse Laplace transform

→ expr3: ilt(expr2,s,t);

expr3 $\frac{\%e^{k\,t}}{2}+\frac{\%e^{-(k\,t)}}{2}$

→ exponentialize(cosh(k·t));

(%o23) $\frac{\%e^{k\,t}+\%e^{-(k\,t)}}{2}$

7 Differential equation with the Laplace transform

→ eq1: diff(y(t),t) – y(t) = exp(–t);

eq1 $\frac{d}{d\,t}\,y(t)-y(t)=\%e^{-t}$

Let's assume the initial condition y(0) = 0

→ eq2: laplace(eq1,t,s);

eq2 $s\ \mathrm{laplace}(y(t),t,s)-\mathrm{laplace}(y(t),t,s)-y(0)=\frac{1}{s+1}$

→ eq3: subst(0,y(0),eq2);

eq3 $s\ \mathrm{laplace}(y(t),t,s)-\mathrm{laplace}(y(t),t,s)=\frac{1}{s+1}$

→ eq4: factor(eq3);

eq4 $(s-1)\ \mathrm{laplace}(y(t),t,s)=\frac{1}{s+1}$

→ eq5: solve(eq4,'laplace(y(t),t,s))[1];

eq5 $\mathrm{laplace}(y(t),t,s)=\frac{1}{s^2-1}$

→ ilt(rhs(eq5),s,t);

(%o29) $\frac{\%e^{t}}{2}-\frac{\%e^{-t}}{2}$