Some (definitely not all!) **orthogonal polynomials and eigenfunctions of the Sturm-Liouville problem**

Polynomial/ Function	**Interval**	$w(x)$	$p(x)$ **(nb 1)**	$s(x)$	λ	**Standard normalization** N^2	**Generating function** **(nb 2)**	K_n **for Rodrigues's formula**
Harmonic $\{e^{inx}\}$	$[-\pi,\pi]$	1	1	0	n^2	π	-	not a polynomial
Legendre $P_l(x)$	$[-1,1]$	1	$1-x^2$	0	$l(l+1)$	$\dfrac{2}{2l+1}$	$(1-2xt+t^2)^{-1/2}$	$\dfrac{1}{2^l l!}\cdot(-1)^l$ **(nb 3)**
Associated Legendre $P_l^m(x)$	$[-1,1]$	1	$1-x^2$	$\dfrac{m^2}{1-x^2}$	$l(l+1)$	$\dfrac{2}{2l+1}\cdot\dfrac{(l+m)!}{(l-m)!}$	Modifications needed. Main definition: $P_l^m(x)=(-1)^m\dfrac{d^m}{dx^m}P_l(x)$	
Bessel of the first kind $J_p(x)$	$[0,1]$	x	x	$-\dfrac{p^2}{x}$	1 **(nb 4)**	$\int_0^1[J_p(ax)]^2x\,dx$ $=\dfrac{1}{2}[J_p'(a)]^2$ (boundary condition $J_p(a)=0$) **(nb 5)**	$e^{(x/2)(t-1/t)}$	not a polynomial
Laguerre $L_n(x)$	$[0,\infty]$	e^{-x}	xe^{-x}	0	n	1	$\dfrac{e^{-xt/(1-t)}}{1-t}$, $a_n=1/n!$	$\dfrac{1}{n!}$
Associated Laguerre $L_n^k(x)$ **(nb 6)**	$[0,\infty]$	x^ke^{-x}	$x^{k+1}e^{-x}$	0	$-n$	$\dfrac{(n+k)!}{n!}$	$\dfrac{e^{-xt/(1-t)}}{(1-t)^{k+1}}$, $a_n=1/n!$	$\dfrac{1}{n!}$
Hermite $H_n(x)$ **(nb 7)**	$[-\infty,\infty]$	e^{-x^2}	e^{-x^2}	0	$2n$	$2^n\pi^{1/2}n!$	e^{-t^2+2tx}, $a_n=1/n!$	$(-1)^n$
Chebyshev polynomials $T_n(x)$	$[-1,1]$	$\dfrac{1}{\sqrt{1-x^2}}$	$(1-x^2)^{\frac{1}{2}}$	0	n^2	$\pi/2$, if $n\neq 0$; π, if $n=0$	$\dfrac{1-tx}{1-2tx+t^2}$	$\dfrac{(-2)^n n!}{(2n)!}$

Checked against: Weber & Arfken (2003) *Essential Mathematical Methods for Physicists*, Academic Press. Chapters 9-13.
The Chebyshev polynomials are taken from Korn & Korn (1968) *Mathematical handbook...* , table 21.7-1.
However: please be careful and always check.

(nb 1) Warning, when computing the Rodrigues's formula: $p_0(x)=p(x)/w(x)$.
(nb 2) Here, $a_n=1$ in the respective series expansion, unless otherwise specified. Warning: generating functions are not unique.
(nb 3) In most books, they take in Rodrigues's formula $p_0(x)\to -p_0(x)$ and therefore they show no factor $(-1)^l$ in K_l. However, they only do it in the Rodrigues's formula and not in the differential equation, so this is why we prefer to modify the K_l and introduce the minus sign there instead.
(nb 4) It is customary to make the change of variables $x=k\rho \implies \dfrac{d}{dx}=\dfrac{1}{k}\dfrac{d}{d\rho}$. After this change of variables, $\lambda=k^2$ and the interval is rescaled to $[0,\alpha]$ where the boundary condition is now $J_p(k\alpha)=0$. Then, the normalization changes to $\int_0^\alpha \rho\,[J_p(k\rho)]^2d\rho=\dfrac{\alpha^2}{2}\left[J_p'(c_{pm})\right]^2$ where c_{pm} are the zeros of the Bessel function.
(nb 5) See also nb 4. Because of recurrence formulae, one can express the normalization in other ways, for example, in terms of $J_\nu'(k_{\mu\nu})=-J_{\nu+1}(k_{\nu\mu})$. The orthogonalization is achieved by considering Dirichlet boundary conditions, i.e., $J_\nu(ka)=0$. The Bessel function is oscillatory, so one needs to choose a zero at the right distance a to satisfy the boundary condition.
(nb 6) Similarly to the case of the associated Legendre polynomials, the Associated Laguerre polynomials are defined in terms of a derivative of the Laguerre polynomials, namely, $L_n^k(x)=(-1)^k\dfrac{d^k}{dx^k}L_{k+n}(x)$.
(nb 7) There is an alternative way of defining the Hermite polynomials and its differential equation. Here we take the "Physicist Hermite polynomials".

Generalized Bessel ODE

The ODE $y'' + \dfrac{1-2a}{x}y' + \left[(bcx^{c-1})^2 + \dfrac{a^2 - p^2c^2}{x^2}\right]y = 0$
with $a, b, c, p \in \mathbb{C}$ has the solution $y(x) = x^a Z_p(bx^c)$, where Z_p is a combination of Bessel $[J_p(x)]$ and Neumann $[Y_p(x) = (\cos(\pi p)J_p(x) - J_{-p}(x))/\sin(\pi p)]$ functions (multiplied by constants).

Equation	**Solutions**	a	b	c	p
Bessel	$J_p(x), Y_p(x)$ $H_p^{(1,2)} = J_p(x) \pm iY_p(x)$	0	1	1	p
Bessel (rescaled)	$J_p(k\rho), Y_p(k\rho), H_p(k\rho)$	0	k	1	p
Modified Bessel	$I_p(x) = i^{-p}J_p(ix)$ $K_p(x) = \frac{\pi}{2}i^{p+1}H_p^{(1)}(ix)$	0	i	1	p
Spherical Bessel	$j_n(x) = \sqrt{\frac{\pi}{2x}}J_{(2n+1)/2}(x)$ $y_n(x) = \sqrt{\frac{\pi}{2x}}Y_{(2n+1)/2}(x)$ $h_n^{(1,2)} = j_n(x) \pm iy_n(x)$	$-1/2$	1	1	$n + \frac{1}{2}, n \in \mathbb{Z}$
Ber, bei, ker, kei functions	$J_0(i^{3/2}x) = \mathrm{ber}\,x + i\,\mathrm{bei}\,x$ $K_0(i^{3/2}x) = \mathrm{ker}\,x + i\,\mathrm{kei}\,x$	0	$i^{3/2}$	1	0